Distributed Time-Varying Formation Robust Tracking for General Linear Multiagent Systems With Parameter Uncertainties and External Disturbances

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***Abstract*—This paper investigates the time-varying formation robust tracking problems for high-order linear multiagent systems with a leader of unknown control input in the presence of heterogeneous parameter uncertainties and external disturbances. The followers need to accomplish an expected time-varying formation in the state space and track the state trajectory produced by the leader simultaneously. First, a time-varying formation robust tracking protocol with a totally distributed form is proposed utilizing the neighborhood state information. With the adaptive updating mechanism, neither any global knowledge about the communication topology nor the upper bounds of the parameter uncertainties, external disturbances and leader's unknown input are required in the proposed protocol. Then, in order to determine the control parameters, an algorithm with four steps is presented, where feasible conditions for the followers to accomplish the expected time-varying formation tracking are provided. Furthermore, based on the Lyapunov-like analysis theory, it is proved that the formation tracking error can converge to zero asymptotically. Finally, the effectiveness of the theoretical results is verified by simulation examples.**

***Index Terms*—Adaptive control, external disturbance, multiagent system, parameter uncertainty, time-varying formation tracking.**

I. Introduction

FORMATION control of multiagent systems has attracted a great quantity of attention from different scientific communities and found potential applications in many practical scenarios, such as cooperative surveillance [1], source seeking [2], fuel flow reduction [3], and so on. The goal of formation control is to make the states of a cluster of agents with interaction capacities form an expected configuration dependent on the mission requirements. Several classic formation approaches, including virtual structure [4], behavior [5], and leader–follower [6] based approaches, have been presented in robotics community. However, these three strategies have their own disadvantages as shown in [7] and [8]. For example, it is difficult to control autonomous agents in formation with a distributed form using the virtual structure approach. The weakness of behavior-based strategy is the difficulty to build the quantitative mathematical model and analyze the stability of the entire system theoretically.

In the past decades, consensus control of multiagent systems has been investigated widely [9]–[21]. Inspired by the consensus theory, distributed control approach using the neighborhood interaction information becomes the research focus of formation control. It is shown in [22] that consensus-based formation control protocol in a distributed form is more general than the above three formation strategies. In [23] and [24], time-invariant formation control protocols for a group of agents with high-order linear dynamics were presented using consensus strategy. However, time-varying formation configurations are appropriate to some applications where the mission requirements and external environment change rapidly. For example, in source seeking applications, the expected formation should be time-varying to follow the source gradient direction. Moreover, it is necessary to transform the shape of a formation in the obstacle avoidance situation. Time-varying formation control approaches for high-order linear multiagent systems were proposed in [25]–[28]. Considering the influence of communication time delays, a time-varying formation protocol and feasibility conditions were given in [25]. Dong *et al.* [26] proposed a group formation control strategy to deal with various time-varying subformations. A time-varying formation approach was presented in [27] for a cluster of agents with switching communication topologies. Wang *et al.* [28] investigated the formation control problems with a totally distributed form by using the adaptive mechanism.

Note that only leaderless formation stabilization control strategies for multiagent systems with high-order dynamics were presented in [23]–[28]. However, besides achieving the desired formation shape, the whole formation should also track the state trajectory produced by the leader in some practical situations, e.g., the cooperative attacking or target enclosing tasks. Thus, it is significant to investigate the formation tracking problems. Different from the classic leader–follower

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formation strategy, only a subset of followers are assumed to receive information from the leader directly and each agent uses neighborhood information to construct distributed control protocols in the consensus-based formation tracking approach. Xiao *et al.* [29] considered formation tracking problems for a cluster of agents with first-order dynamics based on the finite-time consensus strategy. Distributed time-invariant formation tracking approaches using adaptive technique for multirobot systems were proposed in [30] and [31]. Ge *et al.* [32] applied sliding-mode estimators to handle the formation tracking problems for multiple Euler–Lagrange systems. In [33], the communication relationship among followers could be switching, and a time-varying formation tracking protocol for a team of micro quadrotors with second-order dynamics was presented. Using the two-level consensus strategy, time-invariant formation tracking problems for a group of high-order linear agents were investigated in [34]. Dong and Hu [35] presented a time-varying formation tracking protocol for high-order multiagent systems in the presence of multiple leaders. The high-order linear dynamics of every agent is assumed to be totally known and identical in [23]–[28], [34], and [35]. However, it could be too strict in some practical circumstances, and the multiagent systems may suffer from certain unknown parameter uncertainties or external disturbances [36]. Moreover, in the case where the leader is a noncooperative target, all followers cannot get the leader's control input directly, which is usually time-varying and nonzero. How to design time-varying formation robust tracking protocols for uncertain multiagent systems with high-order dynamics and leader's unknown control input is a significant and challenging problem.

Motivated by the facts stated above, this paper investigates the time-varying formation robust tracking problems for multiagent systems with high-order linear dynamics in the presence of heterogeneous parameter uncertainties, external disturbances and leader's unknown input. Compared with the existing works, the main contributions of this paper are threefold. First, the followers can accomplish an expected time-varying formation in the state space and track the state trajectory produced by the leader simultaneously. Only leaderless formation stabilization control problems were considered in [23]–[28]. Formation tracking problems were studied in [29]–[35], but the agents in [29]–[33] have first-order or second-order dynamics and the control protocols in [30], [31], and [34] can only deal with time-invariant formations. Second, the high-order multiagent systems can still accomplish the expected time-varying formation tracking in the case where the followers suffer from heterogeneous parameter uncertainties and external disturbances and the control input of the leader is unknown and time-varying. In [23]–[28], [34], and [35], only totally known high-order linear dynamics was considered and the expected formation could not be kept under the influences of uncertainties and disturbances. The leader's control input is assumed to be equal to zero in [33] and [35], but it may be restrictive and impractical when the leader is noncooperative. Third, the proposed control protocol is determined by each follower with a totally distributed form, which requires neither any global knowledge nor the upper bounds of uncertainties, disturbances and leader's unknown input. The bounds of disturbances are regarded to be known in [37], but it is difficult to accurately determine these bounds in practical applications. These bounds are considered to be unknown and only adaptive analytical estimates are needed in this paper. The time-varying formation protocols in [25]–[27], [33], and [35] rely on the minimum nonzero eigenvalue of the Laplacian matrix, but it is global knowledge since every agent must be aware of the Laplacian matrix. This shortcoming is overcome in this paper by using the adaptive coupling weights.

The rest of this paper is organized as follows. Section II provides some definitions and results about graph theory and gives the problem description. A formation robust tracking protocol and stability analysis are presented in Section III. Section IV gives two numerical examples. The conclusions are drawn in Section V.

Throughout this paper, let $\mathbf{1}_N$ stand for a column vector consisting of 1 with size N. I_n is used to represent an identity matrix with dimension n. A diagonal matrix is denoted by diag$\{\cdot\}$. The superscript T stands for the transpose of real matrices. The two norm of vectors or matrices is denoted by $\|\cdot\|$. For a vector function $g(t) : [0, \infty) \to \mathbb{R}^n$, $g(t) \in \mathbb{L}_2$ stands for $(\int_0^\infty g^T(\tau)g(\tau)\mathrm{d}\tau)^{1/2} < \infty$ and $g(t) \in \mathbb{L}_\infty$ denotes $\sup_{t\geq 0}\|g(t)\| < \infty$. For a matrix $Q \in \mathbb{R}^{n\times n}$, $Q > 0$ represents that Q is a positive definite matrix. $\lambda_{\min}(Q)$ and $\lambda_{\max}(Q)$ stand for its minimum and maximum eigenvalues, respectively.

II. Preliminaries and Problem Description

Basic definitions and results about graph theory and the problem description are given in this section.

A. Basic Graph Theory

A directed graph with N nodes is described by $G = \{S, E, W\}$, where $S = \{s_1, s_2, \ldots, s_N\}$ and $E \subseteq \{(s_i, s_j) : s_i, s_j \in S; i \neq j\}$ are, respectively, the set of nodes and edges, and $W = [w_{ij}] \in \mathbb{R}^{N\times N}$ denotes the adjacency matrix with nonnegative weights w_{ij}. Let $e_{ij} = (s_i, s_j)$ stand for an edge of graph G, where nodes s_i and s_j denote the parent node and the child node, respectively. The weight $w_{ij} > 0$ if and only if $e_{ji} \in E$, and $w_{ij} = 0$ otherwise. The in-degree matrix of G is represented by $D = \mathrm{diag}\{\deg_{in}(s_1), \ldots, \deg_{in}(s_N)\}$, where $\deg_{in}(s_i) = \sum_{j=1}^N w_{ij}$. The Laplacian matrix L is defined as $L = D - W$. Let $N_i = \{s_j \in S : (s_j, s_i) \in E\}$ denote the set of neighbors of node s_i. A graph is said to be undirected if $e_{ij} \in E$ implies $e_{ji} \in E$ and $w_{ij} = w_{ji}$. If there is a root node which has at least one directed path to every other node, the graph is said to contain a spanning tree.

Lemma 1 [10]: If G has a spanning tree, 0 is a simple eigenvalue of $L \in \mathbb{R}^{N\times N}$ with the associated right eigenvector $\mathbf{1}_N$, and the real parts of all the other $N - 1$ eigenvalues are positive.

B. Problem Description

Consider a multiagent system with $N+1$ agents on a graph $\bar{G}$, which is composed of one leader and N followers. The index of leader is represented by 0, and agents labeled by $1, 2, \ldots, N$ are followers.

The dynamics of the leader is described by

$$\dot{x}_0(t) = Ax_0(t) + Bu_0(t) \tag{1a}$$

where $x_0(t) \in \mathbb{R}^n$ is the state and $u_0(t) \in \mathbb{R}^m$ is the control input of the leader. $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$ are constant known matrices with $\text{rank}(B) = m$ and $n \geq m$, which means that each column of B is linearly independent and there are no redundant control input components (see [27, Remark 1] for more details). The leader's state is required to be bounded. The follower i ($i \in \{1, 2, \ldots, N\}$) is assumed to suffer from parameter uncertainties and external disturbances, and the dynamics of each follower is described by

$$\dot{x}_i(t) = (A + \Delta A_i(t))x_i(t) + Bu_i(t) + d_i(t) \tag{1b}$$

where $x_i(t) \in \mathbb{R}^n$ is the state and $u_i(t) \in \mathbb{R}^m$ is the control input of the follower i, $\Delta A_i(t) \in \mathbb{R}^{n\times n}$ denotes the time-varying unknown parameter uncertainty, and $d_i(t) \in \mathbb{R}^n$ represents the time-varying unknown external disturbance.

The general case where the leader's control input $u_0(t)$ can be time-varying unknown and nonzero is considered in this paper, which is more difficult to deal with than $u_0(t) \equiv 0$ in [33] and [35]. $u_0(t)$ is required to satisfy the following mild assumption.

Assumption 1: The leader's control input $u_0(t)$ is bounded, and there exists an unknown positive constant μ such that $\|u_0(t)\| \leq \mu$.

Assume that $\Delta A_i(t)$ and $d_i(t)$ satisfy the following standard matching condition and bounded condition, which are widely used in [36] and [38]–[40].

Assumption 2: There exist matrix $N_i(t)$ and vector $\bar{d}_i(t)$ such that $\Delta A_i(t) = BN_i(t)$ and $d_i(t) = B\bar{d}_i(t)$, $i = 1, 2, \ldots, N$.

Assumption 3: There exist unknown positive constants α_i and γ_i such that $\|N_i(t)\| \leq \alpha_i$ and $\|\bar{d}_i(t)\| \leq \gamma_i$, $i = 1, 2, \ldots, N$.

Let $h_i(t) \in \mathbb{R}^n$ ($i = 1, 2, \ldots, N$) represent piecewise continuously differentiable vectors, then the expected time-varying formation of followers can be specified by a vector $h(t) = [h_1^T(t), h_2^T(t), \ldots, h_N^T(t)]^T$.

Definition 1: For any given bounded initial states, if

$$\lim_{t\to\infty}(x_i(t) - h_i(t) - x_0(t)) = 0, \quad i = 1, 2, \ldots N \tag{2}$$

then the expected time-varying formation tracking is accomplished by multiagent system (1).

Remark 1: From Definition 1, if $h(t) \equiv 0$, then the time-varying formation tracking problem is reducible to the consensus tracking problem. If $h(t)$ is a nonzero constant vector, then Definition 1 becomes the definition for the time-invariant formation tracking. Moreover, if $h(t)$ is chosen to satisfy $\lim_{t\to\infty}\sum_{i=1}^N h_i(t) = 0$, it holds from (2) that $\lim_{t\to\infty}(x_0(t) - \sum_{i=1}^N x_i(t)/N) = 0$, which implies that the target enclosing is accomplished by multiagent system (1). Therefore, consensus tracking, time-invariant formation tracking and target enclosing problems can be unified to the framework of time-varying formation tracking.

The control objective of this paper is to make all followers accomplish an expected time-varying formation and track the state trajectory of the leader simultaneously. This paper mainly pays attention to how to design a robust control protocol with a totally distributed form such that multiagent system (1) can accomplish the formation tracking even with heterogeneous parameter uncertainties, external disturbances and leader's unknown input.

III. Time-Varying Formation Tracking Protocol Design and Analysis

A time-varying formation tracking protocol with a totally distributed form is presented first in this section. Then an algorithm to determine the control parameters is given. Finally, it is proved that formation tracking can be accomplished using the Lyapunov-like analysis theory.

The communication topology with $N+1$ agents is denoted by a directed graph $\bar{G}$, and only a subset of the followers are assumed to receive state information from the leader directly.

Assumption 4: There exists a spanning tree in the graph $\bar{G}$ and the leader is the root node. In addition, the topology among N followers is undirected.

Let $\bar{L}$ represent the Laplacian matrix for the graph $\bar{G}$. According to the leader–follower topology structure, $\bar{L}$ can be described by

$$\bar{L} = \begin{bmatrix} 0 & 0_{1\times N} \\ L_{lf} & L_{ff} \end{bmatrix} \tag{3}$$

where $L_{lf} = [-w_{10}, -w_{20}, \ldots, -w_{N0}]^T$ and $L_{ff} \in \mathbb{R}^{N\times N}$. From Lemma 1 and Assumption 4, one can get that L_{ff} is a symmetric matrix and $L_{ff} > 0$.

For follower i ($i \in \{1, 2, \ldots, N\}$), let $\hat{c}_i(t)$ denote the time-varying coupling weight. The adaptive parameters $\hat{\alpha}_i(t)$ and $\hat{\beta}_i(t)$ stand for the analytical estimates of the upper bounds of parameter uncertainty and the sum of external disturbance and leader's unknown input, i.e., α_i and $\beta_i = \gamma_i + \mu$, respectively. Let $\sigma_i(t) \in \mathbb{R}$ be positive bounded and uniform continuous function such that

$$\lim_{t\to\infty}\int_{t_0}^{t}\sigma_i(\tau)\mathrm{d}\tau \leq \bar{\sigma}_i < \infty \tag{4}$$

where $\bar{\sigma}_i$ denotes a positive scalar. For an expected time-varying formation specified by $h(t)$, let $\delta_i(t)$ represent the formation tracking neighborhood error of the follower i ($i \in \{1, 2, \ldots, N\}$), and $\delta_i(t)$ is defined by

$$\delta_i(t) = w_{i0}(x_i(t) - h_i(t) - x_0(t)) + \sum_{j=1}^{N} w_{ij}\big((x_i(t) - h_i(t)) - (x_j(t) - h_j(t))\big) \tag{5}$$

where $w_{i0} > 0$ if there is an edge e_{0i} from the leader to the follower i in the graph $\bar{G}$ and $w_{i0} = 0$ otherwise.

Let $\omega_i(t) = (\hat{\alpha}_i^2(t)\|x_i(t)\|^2)/(\hat{\alpha}_i(t)\|x_i(t)\|\|B^TP\delta_i(t)\| + \sigma_i(t))$ and $\varphi_i(t) = (\hat{\beta}_i^2(t))/(\hat{\beta}_i(t)\|B^TP\delta_i(t)\| + \sigma_i(t))$. Based on the local neighborhood information, the distributed time-varying formation robust tracking protocol is presented as

$$u_i(t) = -\,v_i(t) - \big(\hat{c}_i(t) + \omega_i(t) + \varphi_i(t)\big)B^TP\delta_i(t) \tag{6}$$

where $v_i(t) \in \mathbb{R}^m$ is the formation tracking compensational input decided by $h(t)$, and $P \in \mathbb{R}^{n\times n}$ is a positive definite matrix.

Remark 2: The formation tracking neighborhood error $\delta_i(t)$ and the compensational input $v_i(t)$ $(i = 1, 2, \ldots, N)$ are applied to construct the distributed tracking protocol (6). The first term in (5) represents the available tracking error for the informed followers which can receive the leader's state information directly. The second term in (5) stands for the formation neighborhood error among followers and is used to drive all followers to accomplish the desired formation configuration.

Let $Y = [\tilde{B}^T, \bar{B}^T]^T$ consisting of $\tilde{B} \in \mathbb{R}^{m\times n}$ and $\bar{B} \in \mathbb{R}^{(n-m)\times n}$ represent a nonsingular matrix, where $\tilde{B}B = I_m$ and $\bar{B}B = 0$. The existence of matrix Y is guaranteed by the condition $\text{rank}(B) = m$. An algorithm is proposed in the following to design the parameters in the robust tracking protocol (6).

Algorithm 1: The following four steps are used to determine the formation tracking protocol (6).

Step 1: For follower i $(i \in \{1, 2, \ldots, N\})$ and an expected time-varying formation specified by $h(t)$, check the time-varying formation tracking feasible condition described by

$$\lim_{t\to\infty} \left\|\bar{B}Ah_i(t) - \bar{B}\dot{h}_i(t)\right\| = 0 \tag{7}$$

where the convergence rate of (7) is required to be faster than $t^{-1/2}$. If the condition (7) is satisfied, then continue; else $h(t)$ is not feasible for multiagent system (1) using the protocol (6) and the algorithm stops.

Step 2: The formation tracking compensational input $v_i(t)$ is calculated by

$$v_i(t) = \tilde{B}\big(Ah_i(t) - \dot{h}_i(t)\big),\ i = 1, 2, \ldots, N. \tag{8}$$

Step 3: Solve the following algebraic Riccati equation (ARE) for a positive definite matrix P:

$$A^TP + PA - PBB^TP + I_n = 0. \tag{9}$$

Step 4: The adaptive parameters $\hat{c}_i(t)$, $\hat{\alpha}_i(t)$ and $\hat{\beta}_i(t)$ $(i = 1, 2, \ldots, N)$ are generated by the following updating laws:

$$\dot{\hat{c}}_i(t) = -\eta_{1i}\sigma_i(t)\hat{c}_i(t) + \eta_{1i}\left\|B^TP\delta_i(t)\right\|^2 \tag{10}$$

$$\dot{\hat{\alpha}}_i(t) = -\eta_{2i}\sigma_i(t)\hat{\alpha}_i(t) + \eta_{2i}\|x_i(t)\|\left\|B^TP\delta_i(t)\right\| \tag{11}$$

$$\dot{\hat{\beta}}_i(t) = -\eta_{3i}\sigma_i(t)\hat{\beta}_i(t) + \eta_{3i}\left\|B^TP\delta_i(t)\right\| \tag{12}$$

where η_{1i}, η_{2i}, and η_{3i} are any positive constants, and the initial values are finite with $\hat{\alpha}_i(t_0) > 0$ and $\hat{\beta}_i(t_0) > 0$.

Remark 3: As pointed out in [27], even for formation stabilization control of high-order multiagent systems, not all time-varying formations can be accomplished due to the restrictions of the dynamics and the interaction topologies. Similarly, the feasibility condition for the followers with high-order linear dynamics to accomplish the time-varying formation tracking is provided in (7), which implies that the expected formations should satisfy the dynamic constraints of each agent. From (7) and (8), one sees that $v_i(t)$ $(i = 1, 2, \ldots, N)$ can expand the feasible formation set by compensating the expected formation vector $h(t)$. Moreover, as shown in [36], there exists a positive definite matrix P to the ARE (9) if and only if (A, B) is stabilizable.

Remark 4: In the formation robust tracking protocol (6), besides the formation tracking neighborhood error $\delta_i(t)$, the absolute state of each follower, i.e., $x_i(t)$, is also needed, which is due to the presence of matching parameter uncertainty $\Delta A_i(t)$. The computation of the minimum nonzero eigenvalue of Laplacian matrix is avoided using the updating law (10). With the analytical estimates $\hat{\alpha}_i(t)$ and $\hat{\beta}_i(t)$ updated by (11) and (12), the upper bounds α_i and β_i can be unknown. Therefore, the robust adaptive tracking protocol (6) is determined by every agent with a totally distributed form, which requires neither any global information nor the upper bounds of uncertainties, disturbances and leader's unknown input.

For follower i $(i \in \{1, 2, \ldots, N\})$, substituting the control protocol (6) into the multiagent system (1), one has

$$\begin{aligned}\dot{x}_i(t) &= (A + BN_i(t))x_i(t) + B\bar{d}_i(t) - Bv_i(t)\\ &\quad - \big(\hat{c}_i(t) + \omega_i(t) + \varphi_i(t)\big)BB^TP\delta_i(t).\end{aligned} \tag{13}$$

Let $x(t) = [x_1^T(t), x_2^T(t), \ldots, x_N^T(t)]^T$, $\bar{d}(t) = [\bar{d}_1^T(t), \bar{d}_2^T(t), \ldots, \bar{d}_N^T(t)]^T$, $v(t) = [v_1^T(t), v_2^T(t), \ldots, v_N^T(t)]^T$, and $\delta(t) = [\delta_1^T(t), \delta_2^T(t), \ldots, \delta_N^T(t)]^T$. The multiagent system (13) is written in the following compact form:

$$\begin{aligned}\dot{x}(t) &= (I_N \otimes A)x(t) + (I_N \otimes B)\Lambda x(t)\\ &\quad + (I_N \otimes B)\bar{d}(t) - (I_N \otimes B)v(t)\\ &\quad - \Big(\Big(\hat{C} + \Omega + \Phi\Big) \otimes BB^TP\Big)\delta(t)\end{aligned} \tag{14}$$

where $\Lambda = \text{diag}\{N_1(t), N_2(t), \ldots, N_N(t)\}$, $\hat{C} = \text{diag}\{\hat{c}_1(t), \hat{c}_2(t), \ldots, \hat{c}_N(t)\}$, $\Omega = \text{diag}\{\ \omega_1(t), \omega_2(t), \ldots, \omega_N(t)\}$, and $\Phi = \text{diag}\{\varphi_1(t), \varphi_2(t), \ldots, \varphi_N(t)\}$.

A corollary of Barbalat lemma which will be used in the proof is given in the following form.

Lemma 2 [41]: For some $p \in [1, \infty)$, if $g(t) \in \mathbb{L}_p \cap \mathbb{L}_\infty$ and $\dot{g}(t) \in \mathbb{L}_\infty$, then it holds that $\lim_{t\to\infty} g(t) = 0$.

The following theorem guarantees that time-varying formation tracking can be accomplished by multiagent system (1).

Theorem 1: If the expected time-varying formation specified by $h(t)$ satisfies the feasible condition (7) and (A, B) is stabilizable, then multiagent system (1) can accomplish the time-varying formation tracking using the fully distributed robust tracking protocol (6) designed in Algorithm 1 in the presence of heterogeneous parameter uncertainties, external disturbances and leader's unknown input.

Proof: Let $\varsigma_i(t)$ represent the time-varying formation tracking error of the follower i $(i = 1, 2, \ldots, N)$, and $\varsigma_i(t) = x_i(t) - h_i(t) - x_0(t)$. Multiagent system (13) can be transformed into

$$\begin{aligned}\dot{\varsigma}_i(t) &= A\varsigma_i(t) + BN_i(t)x_i(t) + B\bar{d}_i(t)\\ &\quad - \big(\hat{c}_i(t) + \omega_i(t) + \varphi_i(t)\big)BB^TP\delta_i(t)\\ &\quad - Bv_i(t) - Bu_0(t) + Ah_i(t) - \dot{h}_i(t).\end{aligned} \tag{15}$$

Let $\varsigma(t) = [\varsigma_1^T(t), \varsigma_2^T(t), \ldots, \varsigma_N^T(t)]^T$, then multiagent system (15) is written in the following compact form:

$$\begin{aligned}\dot{\varsigma}(t) &= (I_N \otimes A)\varsigma(t) + (I_N \otimes B)\Lambda x(t) + (I_N \otimes B)\bar{d}(t)\\ &\quad - (I_N \otimes B)v(t) - \Big(\Big(\hat{C} + \Omega + \Phi\Big) \otimes BB^TP\Big)\delta(t)\\ &\quad - (\mathbf{1}_N \otimes B)u_0(t) + (I_N \otimes A)h(t) - \dot{h}(t).\end{aligned} \tag{16}$$

Consider the following Lyapunov function candidate:

$$V(t)=\varsigma^T(t)\big(L_{ff}\otimes P\big)\varsigma(t)+\sum_{i=1}^{N}\frac{\tilde{c}_i^2(t)}{\eta_{1i}}+\sum_{i=1}^{N}\frac{\tilde{\alpha}_i^2(t)}{\eta_{2i}}+\sum_{i=1}^{N}\frac{\tilde{\beta}_i^2(t)}{\eta_{3i}} \tag{17}$$

where $\tilde{c}_i(t)=\hat{c}_i(t)-c$, $\tilde{\alpha}_i(t)=\hat{\alpha}_i(t)-\alpha_i$ and $\tilde{\beta}_i(t)=\hat{\beta}_i(t)-\beta_i$. c stands for a positive constant to be determined later. β_i denotes the sum of the upper bounds of external disturbance and leader's unknown input, i.e., $\beta_i=\gamma_i+\mu$.

The time derivative of $V(t)$ along the trajectory of (16) can be obtained as

$$\dot{V}(t)=2\varsigma^T(t)\big(L_{ff}\otimes P\big)\dot{\varsigma}(t)+2\sum_{i=1}^{N}\frac{\tilde{c}_i(t)\dot{\tilde{c}}_i(t)}{\eta_{1i}}+2\sum_{i=1}^{N}\frac{\tilde{\alpha}_i(t)\dot{\tilde{\alpha}}_i(t)}{\eta_{2i}}+2\sum_{i=1}^{N}\frac{\tilde{\beta}_i(t)\dot{\tilde{\beta}}_i(t)}{\eta_{3i}}. \tag{18}$$

Substituting (16) and the adaptive updating laws (10)–(12) into (18) yields

$$\begin{aligned}\dot{V}(t)&=2\varsigma^T(t)\big(L_{ff}\otimes PA\big)\varsigma(t)+2\varsigma^T(t)\big(L_{ff}\otimes PB\big)\Lambda x(t)\\&\quad+2\varsigma^T(t)\big(L_{ff}\otimes PB\big)\bar{d}(t)-2\varsigma^T(t)\big(L_{ff}\otimes PB\big)v(t)\\&\quad-2\varsigma^T(t)\Big(L_{ff}\Big(\hat{C}+\Omega+\Phi\Big)\otimes PBB^TP\Big)\delta(t)\\&\quad+2\varsigma^T(t)\big(\big(L_{ff}\otimes PA\big)h(t)-\big(L_{ff}\otimes P\big)\dot{h}(t)\big)\\&\quad-2\varsigma^T(t)\big(L_{ff}\mathbf{1}_N\otimes PB\big)u_0(t)\\&\quad+2\sum_{i=1}^{N}\tilde{c}_i(t)\Big(-\hat{c}_i(t)\sigma_i(t)+\big\|B^TP\delta_i(t)\big\|^2\Big)\\&\quad+2\sum_{i=1}^{N}\tilde{\alpha}_i(t)\big(-\hat{\alpha}_i(t)\sigma_i(t)+\|x_i(t)\|\big\|B^TP\delta_i(t)\big\|\big)\\&\quad+2\sum_{i=1}^{N}\tilde{\beta}_i(t)\Big(-\hat{\beta}_i(t)\sigma_i(t)+\big\|B^TP\delta_i(t)\big\|\Big).\end{aligned} \tag{19}$$

From the definitions of $\delta_i(t)$ and $\varsigma_i(t)$ $(i=1,2,\ldots,N)$, one gets that $\delta(t)=(L_{ff}\otimes I_n)\varsigma(t)$. It holds from the definitions of $\hat{C}$ and $\tilde{c}_i(t)$ that

$$\begin{aligned}&-2\varsigma^T(t)\Big(L_{ff}\hat{C}\otimes PBB^TP\Big)\delta(t)+2\sum_{i=1}^{N}\tilde{c}_i(t)\big\|B^TP\delta_i(t)\big\|^2\\&\quad=-2\sum_{i=1}^{N}\hat{c}_i(t)\big\|B^TP\delta_i(t)\big\|^2+2\sum_{i=1}^{N}\big(\hat{c}_i(t)-c\big)\big\|B^TP\delta_i(t)\big\|^2\\&\quad=-2c\varsigma^T(t)\Big(L_{ff}^2\otimes PBB^TP\Big)\varsigma(t).\end{aligned} \tag{20}$$

Since Assumption 1 holds, one has

$$\begin{aligned}-2\varsigma^T(t)\big(L_{ff}\mathbf{1}_N\otimes PB\big)u_0(t)&=-2\sum_{i=1}^{N}\delta_i^T(t)PBu_0(t)\\&\le 2\sum_{i=1}^{N}\|u_0(t)\|\big\|B^TP\delta_i(t)\big\|\\&\le 2\mu\sum_{i=1}^{N}\big\|B^TP\delta_i(t)\big\|.\end{aligned} \tag{21}$$

From Assumption 3, one can obtain

$$\begin{aligned}2\varsigma^T(t)\big(L_{ff}\otimes PB\big)\Lambda x(t)&=2\sum_{i=1}^{N}\delta_i^T(t)PBN_i(t)x_i(t)\\&\le 2\sum_{i=1}^{N}\alpha_i\big\|B^TP\delta_i(t)\big\|\|x_i(t)\|\end{aligned} \tag{22}$$

$$2\varsigma^T(t)\big(L_{ff}\otimes PB\big)\bar{d}(t)\le 2\sum_{i=1}^{N}\gamma_i\big\|B^TP\delta_i(t)\big\|. \tag{23}$$

Substitute (20)–(23) into (19). Since $\hat{\alpha}_i(t)=\tilde{\alpha}_i(t)+\alpha_i$ and $\hat{\beta}_i(t)=\tilde{\beta}_i(t)+\gamma_i+\mu$, one has

$$\begin{aligned}\dot{V}(t)&\le 2\varsigma^T(t)\big(L_{ff}\otimes PA\big)\varsigma(t)-2c\varsigma^T(t)\Big(L_{ff}^2\otimes PBB^TP\Big)\varsigma(t)\\&\quad-2\varsigma^T(t)\big(L_{ff}(\Omega+\Phi)\otimes PBB^TP\big)\delta(t)\\&\quad+2\sum_{i=1}^{N}\hat{\alpha}_i(t)\|x_i(t)\|\big\|B^TP\delta_i(t)\big\|+2\sum_{i=1}^{N}\hat{\beta}_i(t)\big\|B^TP\delta_i(t)\big\|\\&\quad+2\varsigma^T(t)\big(\big(L_{ff}\otimes PA\big)h(t)-\big(L_{ff}\otimes P\big)\dot{h}(t)\big)\\&\quad-2\varsigma^T(t)\big(L_{ff}\otimes PB\big)v(t)-2\sum_{i=1}^{N}\tilde{c}_i(t)\hat{c}_i(t)\sigma_i(t)\\&\quad-2\sum_{i=1}^{N}\tilde{\alpha}_i(t)\hat{\alpha}_i(t)\sigma_i(t)-2\sum_{i=1}^{N}\tilde{\beta}_i(t)\hat{\beta}_i(t)\sigma_i(t).\end{aligned} \tag{24}$$

From the definitions of Ω and $\omega_i(t)$, it holds that

$$\begin{aligned}&-2\varsigma^T(t)\big(L_{ff}\Omega\otimes PBB^TP\big)\delta(t)+2\sum_{i=1}^{N}\hat{\alpha}_i(t)\|x_i(t)\|\big\|B^TP\delta_i(t)\big\|\\&\quad=2\sum_{i=1}^{N}\frac{\hat{\alpha}_i(t)\|x_i(t)\|\big\|B^TP\delta_i(t)\big\|\sigma_i(t)}{\hat{\alpha}_i(t)\|x_i(t)\|\big\|B^TP\delta_i(t)\big\|+\sigma_i(t)}\\&\quad\le 2\sum_{i=1}^{N}\sigma_i(t).\end{aligned} \tag{25}$$

Similarly, according to the definitions of Φ and $\varphi_i(t)$, one can obtain

$$\begin{aligned}&-2\varsigma^T(t)\big(L_{ff}\Phi\otimes PBB^TP\big)\delta(t)+2\sum_{i=1}^{N}\hat{\beta}_i(t)\big\|B^TP\delta_i(t)\big\|\\&\quad=2\sum_{i=1}^{N}\frac{\hat{\beta}_i(t)\big\|B^TP\delta_i(t)\big\|\sigma_i(t)}{\hat{\beta}_i(t)\big\|B^TP\delta_i(t)\big\|+\sigma_i(t)}\\&\quad\le 2\sum_{i=1}^{N}\sigma_i(t).\end{aligned} \tag{26}$$

Let $\tilde{h}(t)=(L_{ff}\otimes A)h(t)-(L_{ff}\otimes I_n)\dot{h}(t)-(L_{ff}\otimes B)v(t)$. Note that $\hat{c}_i(t)=\tilde{c}_i(t)+c$. Then it holds that $-\tilde{c}_i(t)\hat{c}_i(t)=-\tilde{c}_i^2(t)-\tilde{c}_i(t)c\le(1/4)c^2$. In the same way, one can obtain $-\tilde{\alpha}_i(t)\hat{\alpha}_i(t)\le(1/4)\alpha_i^2$ and $-\tilde{\beta}_i(t)\hat{\beta}_i(t)\le(1/4)\beta_i^2$. Utilizing these three inequalities, it follows from (24)–(26) that:

$$\begin{aligned}\dot{V}(t)&\le 2\varsigma^T(t)\big(L_{ff}\otimes PA\big)\varsigma(t)-2c\varsigma^T(t)\Big(L_{ff}^2\otimes PBB^TP\Big)\varsigma(t)\\&\quad+2\varsigma^T(t)(I_N\otimes P)\tilde{h}(t)+\sum_{i=1}^{N}k_i\sigma_i(t)\end{aligned} \tag{27}$$

where k_i is a positive constant and $k_i=4+(c^2/2)+(\alpha_i^2/2)+(\beta_i^2/2)$, $i=1,2,\ldots,N$.

According to the well-known Young's inequality, one has

$$\begin{aligned}2\varsigma^T(t)(I_N\otimes P)\tilde{h}(t) &\le 2\lambda_{\max}(P)\|\varsigma(t)\|\left\|\tilde{h}(t)\right\| \\ &\le \lambda_{\max}(P)\left(\varepsilon\varsigma^T(t)\varsigma(t)+\frac{1}{\varepsilon}\tilde{h}^T(t)\tilde{h}(t)\right)\end{aligned} \tag{28}$$

where ε is a positive constant which can be freely chosen. Substituting (28) into (27) yields

$$\begin{aligned}\dot{V}(t) \le\ & 2\varsigma^T(t)\big(L_{ff}\otimes PA\big)\varsigma(t) - 2c\varsigma^T(t)\Big(L_{ff}^2\otimes PBB^TP\Big)\varsigma(t) \\ &+ \varepsilon\lambda_{\max}(P)\varsigma^T(t)\varsigma(t)+\frac{\lambda_{\max}(P)}{\varepsilon}\tilde{h}^T(t)\tilde{h}(t)+\sum_{i=1}^N k_i\sigma_i(t).\end{aligned} \tag{29}$$

Let λ_i $(i = 1, 2, \ldots, N)$ represent the eigenvalues of L_{ff}. It holds from Lemma 1 and Assumption 4 that $L_{ff} > 0$. Choose a unitary matrix $U \in \mathbb{R}^{N\times N}$ such that $U^T L_{ff} U = \operatorname{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_N\} \triangleq J_L$. Let $\psi(t) = [\psi_1^T(t), \psi_2^T(t), \ldots, \psi_N^T(t)]^T$ and $\psi(t) = (U^T \otimes I_n)\varsigma(t)$, then it holds from (29) that

$$\begin{aligned}\dot{V}(t) \le\ & \psi^T(t)[J_L\otimes\left(A^TP+PA\right)-2cJ_L^2\otimes PBB^TP \\ &+ \varepsilon\lambda_{\max}(P)I_{Nn}]\psi(t)+\frac{\lambda_{\max}(P)}{\varepsilon}\tilde{h}^T(t)\tilde{h}(t)+\sum_{i=1}^N k_i\sigma_i(t) \\ =& -\sum_{i=1}^N\lambda_i\psi_i^T(t)Q_i\psi_i(t)+\frac{\lambda_{\max}(P)}{\varepsilon}\left\|\tilde{h}(t)\right\|^2+\sum_{i=1}^N k_i\sigma_i(t)\end{aligned} \tag{30}$$

where $-Q_i = A^TP + PA - 2c\lambda_i PBB^TP + \varepsilon(\lambda_{\max}(P)/\lambda_i)I_n$, $i = 1, 2, \ldots, N$.

Choose sufficiently large c such that $c > (1/(2\lambda_{\min}(L_{ff})))$, and design sufficiently small ε such that $\varepsilon < (\lambda_{\min}(L_{ff})/\lambda_{\max}(P))$. It follows from ARE (9) that $-Q_i < A^TP + PA - PBB^TP + I_n = 0$, which means that Q_i is a positive definite matrix. From (30), one gets

$$\dot{V}(t) \le -\underline{\lambda}\|\psi(t)\|^2+\frac{\lambda_{\max}(P)}{\varepsilon}\left\|\tilde{h}(t)\right\|^2+\sum_{i=1}^N k_i\sigma_i(t) \tag{31}$$

where $\underline{\lambda} = \lambda_{\min}(L_{ff})\min_{i=1,\ldots,N}(\lambda_{\min}(Q_i))$. It holds from (31) that

$$\begin{aligned}V(t) \le\ & V(t_0)-\int_{t_0}^t\underline{\lambda}\|\psi(\tau)\|^2\mathrm{d}\tau+\frac{\lambda_{\max}(P)}{\varepsilon}\int_{t_0}^t\left\|\tilde{h}(\tau)\right\|^2\mathrm{d}\tau \\ &+\int_{t_0}^t\sum_{i=1}^N k_i\sigma_i(\tau)\mathrm{d}\tau \\ \le\ & V(t_0)+\frac{\lambda_{\max}(P)}{\varepsilon}\int_{t_0}^t\left\|\tilde{h}(\tau)\right\|^2\mathrm{d}\tau+\int_{t_0}^t\sum_{i=1}^N k_i\sigma_i(\tau)\mathrm{d}\tau.\end{aligned} \tag{32}$$

Since the expected formation vector $h(t)$ satisfies the formation tracking feasible condition (7), for follower i $(i = 1, 2, \ldots, N)$, it follows that:

$$\lim_{t\to\infty}\left\|\bar{B}Ah_i(t)-\bar{B}\dot{h}_i(t)-\bar{B}Bv_i(t)\right\| = 0. \tag{33}$$

From (8), one gets

$$\tilde{B}Ah_i(t)-\tilde{B}\dot{h}_i(t)-\tilde{B}Bv_i(t) = 0. \tag{34}$$

Since $Y = [\tilde{B}^T, \bar{B}^T]^T$ is a nonsingular matrix, one can obtain from (33) and (34) that

$$\lim_{t\to\infty}\left\|Ah_i(t)-\dot{h}_i(t)-Bv_i(t)\right\| = 0 \tag{35}$$

which means that $\lim_{t\to\infty}\|\tilde{h}(t)\| = 0$ and its convergence rate is greater than $t^{-1/2}$. Thus the infinite integral of $\|\tilde{h}(t)\|^2$ converges, i.e., $\lim_{t\to\infty}\int_{t_0}^t\|\tilde{h}(\tau)\|^2\mathrm{d}\tau \le \bar{h} < \infty$, where $\bar{h}$ is a positive constant. Together with (4), it holds from (32) that

$$V(t) \le V(t_0)+\frac{\lambda_{\max}(P)}{\varepsilon}\bar{h}+\sum_{i=1}^N k_i\bar{\sigma}_i. \tag{36}$$

Then, one gets that $\psi_i(t)$, $\tilde{c}_i(t)$, $\tilde{\alpha}_i(t)$, and $\tilde{\beta}_i(t)$ $(i = 1, 2, \ldots, N)$ are uniformly bounded.

Furthermore, since $V(t) \ge 0$, it follows from (32) and (36) that:

$$\int_{t_0}^t\underline{\lambda}\|\psi(\tau)\|^2\mathrm{d}\tau \le V(t_0)+\frac{\lambda_{\max}(P)}{\varepsilon}\bar{h}+\sum_{i=1}^N k_i\bar{\sigma}_i \tag{37}$$

which implies that $\psi(t) \in \mathbb{L}_2\cap\mathbb{L}_\infty$. Note the fact that $\varsigma(t) = (U\otimes I_n)\psi(t)$, one has that $\varsigma(t) \in \mathbb{L}_2\cap\mathbb{L}_\infty$. It holds from (16) that $\dot{\varsigma}(t) \in \mathbb{L}_\infty$. According to the Lemma 2, one can obtain that $\lim_{t\to\infty}\|\varsigma(t)\| = 0$, i.e., $\lim_{t\to\infty}(x_i(t) - h_i(t) - x_0(t)) = 0$, $i = 1, 2, \ldots, N$. From Definition 1, one gets that multiagent system (1) can accomplish the time-varying formation tracking using the robust adaptive control protocol (6) with a totally distributed form. This completes the proof. ■

Remark 5: If $\Delta A_i(t) = 0$ and $d_i(t) = 0$ $(i = 1, 2, \cdots, N)$, the problems discussed in this paper become the time-varying formation tracking problems for multiagent systems with identical nominal linear dynamics and leader's unknown input. If $u_0(t) = 0$, the problems turn into the formation robust tracking problems for a group of uncertain agents without leader's control input. Through a simple transformation, Theorem 1 can be utilized to handle these two time-varying formation tracking problems, which have not been investigated extensively in the existing works. In addition, if $h(t) \equiv 0$, the time-varying formation tracking problems reduce to the consensus tracking problems, and all results in this paper are applicable to solve the robust consensus cases discussed in [36].

Remark 6: The positive bounded functions $\sigma_i(t)$ $(i = 1, 2, \ldots, N)$ are applied to the adaptive updating laws (10)–(12). In practical applications, $\sigma_i(t)$ can be chosen as $\sigma_i(t) = \kappa_i e^{-\nu_i t}$, where κ_i and ν_i are positive constants. The improved adaptive laws with σ-functions can make the formation tracking error converge to zero asymptotically. Moreover, compared with the monotonically increasing updating laws in [28], the σ-modification method in this paper can avoid high gain efficiently.

IV. Numerical Simulations

In order to verify the theoretical results, two numerical examples are presented in this section. A third-order multiagent system is considered in Example 1. The proposed

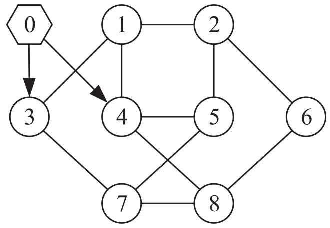


Fig. 1. Interaction topology $\bar{G}_1$.

approach is applied to a multivehicle system and the comparison results with the algorithm in [33] are provided in Example 2.

Example 1: Consider a multiagent system with nine agents. The graph $\bar{G}_1$ with 0-1 weights is shown in Fig. 1. Agent 0 is regarded as the leader and agents labeled by $1, 2, \ldots, 8$ are the followers. The dynamics of each agent is denoted by (1) with $x_i(t) = [x_{i1}(t), x_{i2}(t), x_{i3}(t)]^T$, $u_i(t) = [u_{i1}(t), u_{i2}(t)]^T$ and

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -6 & -4 \end{bmatrix},\ B = \begin{bmatrix} 0 & 1 \\ 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

The leader's control input is set to be $u_0(t) = [1, 0.5\sin(t)]^T$ ($t \le 30$ s), which is unknown to all followers and satisfies Assumption 1. These eight followers are assumed to suffer from heterogeneous parameter uncertainties and external disturbances as shown in Table I. It can be verified that $\Delta A_i(t)$ and $d_i(t)$ ($i = 1, 2, \ldots, 8$) satisfy Assumptions 2 and 3.

All followers need to accomplish a time-varying regular octagon formation tracking, and the expected formation $h(t) = [h_1^T(t), h_2^T(t), \ldots, h_8^T(t)]^T$ is described as

$$h_i(t) = \begin{bmatrix} r\sin\left(\varpi t + \frac{(i-1)\pi}{4}\right) \\ r\varpi\cos\left(\varpi t + \frac{(i-1)\pi}{4}\right) \\ -r\varpi^2\sin\left(\varpi t + \frac{(i-1)\pi}{4}\right) \end{bmatrix},\ i = 1, 2, \ldots, 8.$$

For simplicity, the parameters r and ϖ are chosen as $r = 1$ and $\varpi = 1$. If $h(t)$ is accomplished, these eight followers locate, respectively, at the eight vertexes of a regular octagon and rotate at the angular speed of 1 rad/s while tracking the state trajectory of the leader.

Let $\tilde{B} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and $\bar{B} = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$ such that $\tilde{B}B = I_2$ and $\bar{B}B = 0$. One can obtain that the time-varying formation tracking feasible condition (7) is satisfied. Based on Algorithm 1, the time-varying formation tracking compensational input $v_i(t)$ ($i = 1, 2, \ldots, 8$) and the matrix P can be obtained as

$$v_i = \left[-5\cos\left(t + \left(\frac{(i-1)\pi}{4}\right)\right), 0\right]^T$$
$$P = \begin{bmatrix} 1.4060 & 1.1559 & 0.1155 \\ 1.1559 & 1.9389 & 0.2208 \\ 0.1155 & 0.2208 & 0.1750 \end{bmatrix}.$$

For follower i ($i = 1, 2, \ldots, 8$), let $\eta_{1i} = 1$, $\eta_{2i} = 1$, $\eta_{3i} = 1$, $\sigma_i(t) = 2e^{-2t}$, $\hat{c}_i(0) = 0$, $\hat{\alpha}_i(0) = 5$ and $\hat{\beta}_i(0) = 5$. The initial states of each agent are set to be $x_{ij}(0) = 2(\vartheta - 0.5)$, $i = 0, 1, 2, \ldots, 8$, $j = 1, 2, 3$, where ϑ stands for a random number within the interval [0, 1].

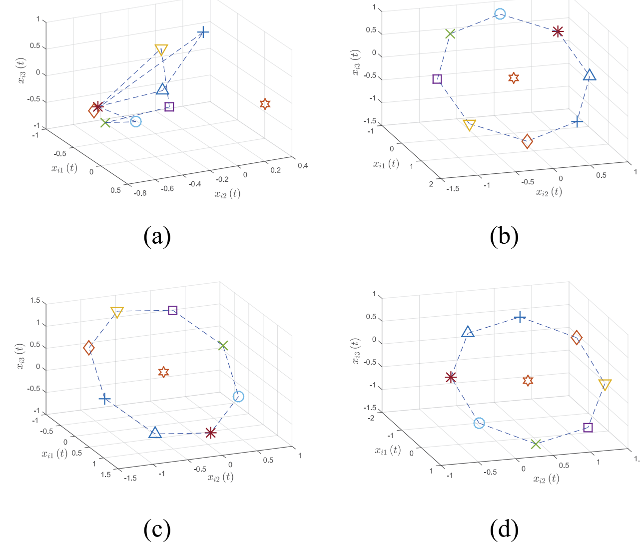


Fig. 2. State snapshots of the eight followers and leader at different time instants. (a) $t = 0$ s. (b) $t = 26$ s. (c) $t = 28$ s. (d) $t = 30$ s.

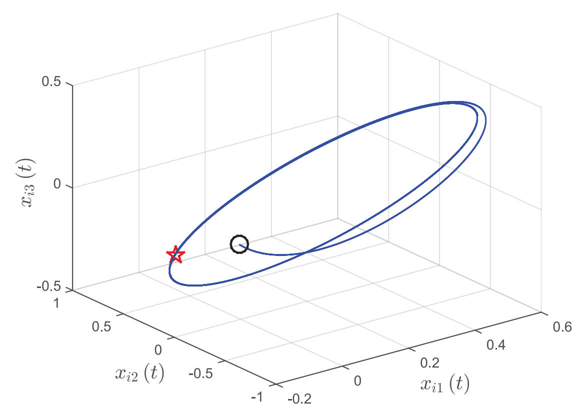


Fig. 3. Trajectory of the leader $x_0(t)$.

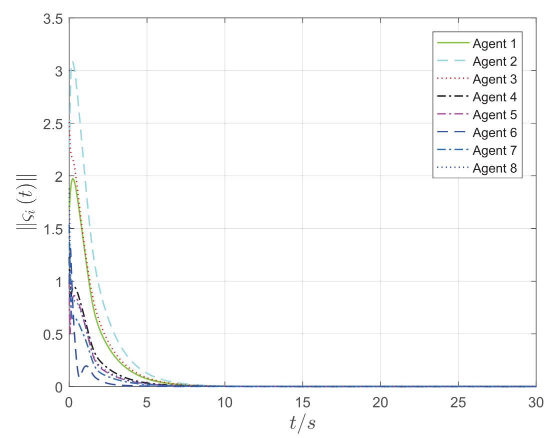


Fig. 4. Curves of the time-varying formation tracking error of each follower.

Fig. 2 depicts the state snapshots of the eight followers and the leader at different time instants, where these eight followers are represented by the plus sign, diamond, downward-pointing triangle, square, cross, circle, asterisk, and upward-pointing triangle, respectively, and the leader is

TABLE I
PARAMETER UNCERTAINTIES AND EXTERNAL DISTURBANCES OF FOLLOWERS

No.	Parameter uncertainties	External disturbances
Agent 1	$\Delta A_1(t)=\begin{bmatrix}0 & 0.5\cos(t) & 0\\ 0 & -0.5\cos(t) & 0\\ \sin(t) & 0 & 0\end{bmatrix}$	$d_1(t)=[0.5,-0.5,\sin(t)]^T$
Agent 2	$\Delta A_2(t)=0$	$d_2(t)=[\cos(t),-\cos(t),0]^T$
Agent 3	$\Delta A_3(t)=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & e^{-0.1t}\end{bmatrix}$	$d_3(t)=[\sin(t),-\sin(t),\cos(t)]^T$
Agent 4	$\Delta A_4(t)=0$	$d_4(t)=[e^{-0.1t},-e^{-0.1t},1]^T$
Agent 5	$\Delta A_5(t)=\begin{bmatrix}0.5\sin(t) & 0 & -\cos(t)\\ -0.5\sin(t) & 0 & \cos(t)\\ 0 & 0 & 0\end{bmatrix}$	$d_5(t)=[\cos(t),-\cos(t),2e^{-0.2t}]^T$
Agent 6	$\Delta A_6(t)=0$	$d_6(t)=[\sin(t),-\sin(t),-1]^T$
Agent 7	$\Delta A_7(t)=\begin{bmatrix}-0.5e^{-0.2t} & 0 & 0\\ 0.5e^{-0.2t} & 0 & 0\\ 0 & 0 & -\sin(t)\end{bmatrix}$	$d_7(t)=[0,0,-\cos(t)]^T$
Agent 8	$\Delta A_8(t)=0$	$d_8(t)=[-0.5,0.5,e^{-0.1t}]^T$

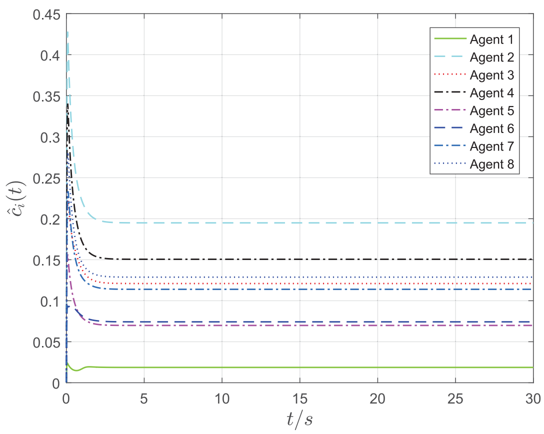


Fig. 5. History of the time-varying coupling weights $\hat{c}_i(t)$.

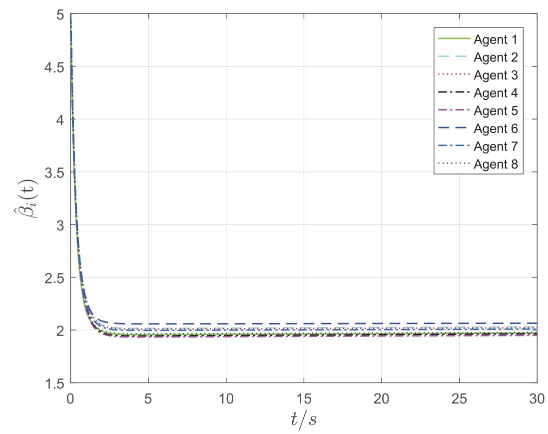


Fig. 7. History of the adaptive updating parameters $\hat{\beta}_i(t)$.

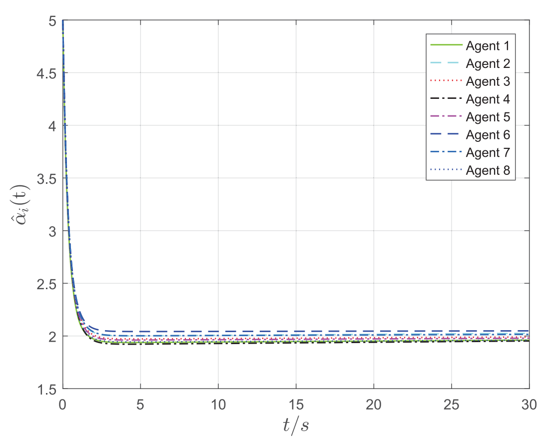


Fig. 6. History of the adaptive updating parameters $\hat{\alpha}_i(t)$.

denoted by the six-pointed star. Fig. 3 shows the state trajectory of the leader within 30 s, where the initial state $x_0(0)$ is denoted by the circle and the final state $x_0(30)$ is represented by the five-pointed star. The two norm of the time-varying formation tracking error, i.e., $\|\varsigma_i(t)\|$ $(i = 1, 2, \ldots, 8)$, is shown in Fig. 4. The adaptive parameters $\hat{c}_i(t)$, $\hat{\alpha}_i(t)$, and $\hat{\beta}_i(t)$ $(i = 1, 2, \ldots, 8)$ are shown in Figs. 5–7. From Figs. 2–4, one sees that the followers achieve the regular octagon formation tracking and the leader's state $x_0(t)$ lies in the center of the octagon. Moreover, the achieved octagon formation rotates around the leader and the edge length is time-varying. As shown in Figs. 5–7, the improved adaptive laws with σ-functions can make the estimate values decrease and converge to finite steady-state values finally. Therefore, the time-varying regular octagon formation robust tracking specified by $h(t)$ is accomplished.

Example 2: Consider a multivehicle system with five vehicles. The graph $\bar{G}_2$ with 0-1 weights is shown in Fig. 8. The vehicle labeled by 0 is the leader and vehicles 1–4 represent the followers. These vehicles move in the horizontal plane, i.e., *X-Y* plane. The multivehicle system needs to carry out a target enclosing task, which is a special case of formation tracking problem. The leader is assumed to be a noncooperative target and all followers cannot get the leader's control input directly. The four followers need to accomplish a time-varying circular formation to enclose the leader.

The dynamics of vehicle i $(i = 0, 1, \ldots, 4)$ is denoted by

$$\begin{cases}\dot{x}_{Xi}(t) = V_i(t)\cos(\phi_i(t))\\ \dot{x}_{Yi}(t) = V_i(t)\sin(\phi_i(t))\\ \dot{\phi}_i(t) = q_i(t)\end{cases}$$

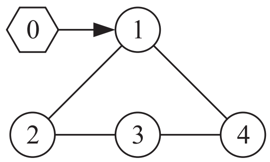


Fig. 8. Interaction topology $\bar{G}_2$.

where $x_{Xi}(t)$ and $x_{Yi}(t)$ are the positions along the x-axis and y-axis, $V_i(t)$ is the linear velocity, $\phi_i(t)$ denotes the heading angle and $q_i(t)$ represents the angular velocity of vehicle i. In light of the dynamic feedback linearization approach in [42], one can obtain the following linearized model of each vehicle:

$$\dot{\chi}_i(t) = \left(I_2 \otimes \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right)\chi_i(t) + \left(I_2 \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)u_i(t)$$

where $\chi_i(t) = [x_{Xi}(t), v_{Xi}(t), x_{Yi}(t), v_{Yi}(t)]^T$ and $u_i(t) = [u_{Xi}(t), u_{Yi}(t)]^T$. $v_{Xi}(t)$ and $v_{Yi}(t)$ are the velocities, and $u_{Xi}(t)$ and $u_{Yi}(t)$ denote the control inputs along the x-axis and y-axis. The unknown control input of the leader is set to be $u_0(t) = [0.03, 0.02]^T$ ($t \leq 30$ s), which satisfies Assumption 1. The vehicle 3 is assumed to suffer from time-varying parameter uncertainties with

$$\Delta A_{X3}(t) = \begin{bmatrix} 0 & 0 \\ 0 & 0.1\cos(t) \end{bmatrix}$$

$$\text{and } \Delta A_{Y3}(t) = \begin{bmatrix} 0 & 0 \\ 0 & 0.1\sin(t) \end{bmatrix}.$$

The vehicles 1–3 are subject to the following heterogeneous external disturbances: $d_{X1}(t) = [0, \cos(t)]^T$, $d_{Y1}(t) = [0, \sin(t)]^T$, $d_{X2}(t) = [0, -e^{-0.1t}]^T$, $d_{Y2}(t) = [0, 0.5e^{-0.2t}]^T$, $d_{X3}(t) = [0, 0.5]^T$, and $d_{Y3}(t) = [0, -1]^T$. It can be verified that $\Delta A_{Xi}(t)$, $\Delta A_{Yi}(t)$, $d_{Xi}(t)$ and $d_{Yi}(t)$ ($i = 1, 2, 3, 4$) satisfy Assumptions 2 and 3.

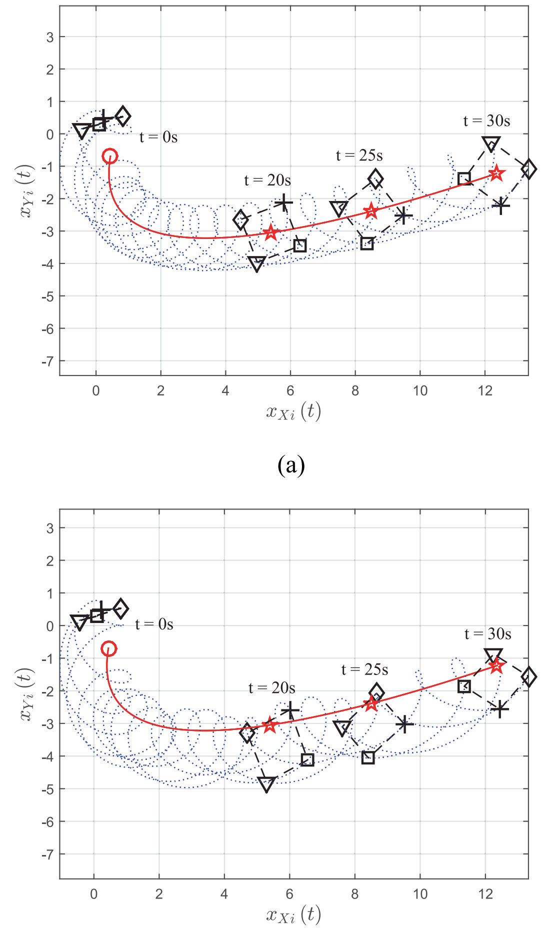
The four followers need to accomplish a time-varying circular formation tracking, and the expected formation is described as

$$h_i(t) = \begin{bmatrix} \cos\left(t + \frac{(i-1)\pi}{2}\right) \\ -\sin\left(t + \frac{(i-1)\pi}{2}\right) \\ \sin\left(t + \frac{(i-1)\pi}{2}\right) \\ \cos\left(t + \frac{(i-1)\pi}{2}\right) \end{bmatrix}, \quad i = 1, 2, 3, 4.$$

When $h(t)$ is accomplished, the followers will locate at the four vertexes of a square and rotate around the leader. Let $\tilde{B} = I_2 \otimes [0, 1]$ and $\bar{B} = I_2 \otimes [1, 0]$. One can obtain that the formation tracking feasible condition (7) is satisfied. From (8), one can get $v_i(t) = [\cos(t + ((i-1)\pi/2)), \sin(t + ((i-1)\pi/2))]^T$, $i = 1, 2, 3, 4$. Solving the ARE (9) gives

$$P = I_2 \otimes \begin{bmatrix} 1.7321 & 1 \\ 1 & 1.7321 \end{bmatrix}.$$

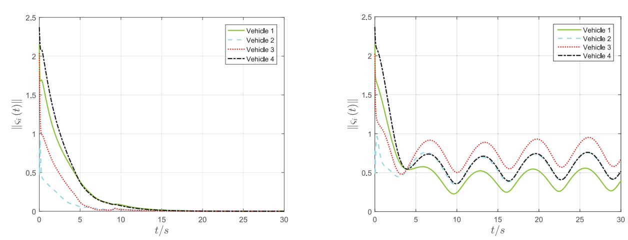
Let $\eta_{1i} = 1$, $\eta_{2i} = 1$, $\eta_{3i} = 1$, and $\sigma_i(t) = e^{-0.5t}$. The initial values of adaptive parameters are set to be $\hat{c}_i(0) = 0$, $\hat{\alpha}_i(0) = 5$ and $\hat{\beta}_i(0) = 5$ ($i = 1, 2, 3, 4$). Let the initial states of each vehicle be $\chi_{ij}(0) = 2(\vartheta - 0.5)$, $i = 0, 1, \ldots, 4$, $j = 1, 2, 3, 4$, where ϑ stands for a random number within the interval [0, 1]. In order to further illustrate the effectiveness of the Algorithm 1, the formation tracking protocol (3) in [33] is provided as a comparison, where the damping constants $\alpha_x = 0$ and $\alpha_v = 0$, the gain matrix $K = I_2 \otimes [-3, -5.1962]$, and other simulation conditions are the same as above.



(a)

(b)

Fig. 9. Position trajectories within $t = 30$ s and snapshots at different time instants of the five vehicles. (a) Approach in the current paper. (b) Approach in [33].



(a) (b)

Fig. 10. Curves of the time-varying formation tracking errors of each follower. (a) Approach in the current paper. (b) Approach in [33].

Fig. 9 shows the position trajectories within $t = 30$s and the position snapshots at different time instants of the five vehicles using the approaches in this paper and in [33], respectively. The trajectories of the leader and followers are denoted, respectively, by red solid line and blue dotted line. The four followers are represented by plus sign, diamond, downward-pointing triangle and square, and the leader is denoted by circle or five-pointed star at different time instants. Fig. 10 depicts the two norm of the time-varying formation tracking errors of each follower. From Figs. 9 and 10, one sees that the four followers accomplish the expected time-varying circular formation to enclose the leader using the approach in this paper. However, the formation tracking cannot be achieved by the multivehicle system using the approach in [33] due to the leader's unknown control input and followers' parameter uncertainties and external disturbances. Therefore, the algorithm in [33] cannot be used to solve the formation robust tracking problems discussed in this paper.

V. Conclusion

Time-varying formation robust tracking problems for a group of agents with high-order linear dynamics in the presence of parameter uncertainties, external disturbances and leader's unknown control input were studied. Using the neighborhood state information, a robust tracking protocol with a totally distributed form was presented. With the adaptive mechanism, the protocol does not require any global knowledge about the communication topology or the upper bounds of the parameter uncertainties, external disturbances and leader's unknown input. Feasible conditions for all followers to accomplish the expected time-varying formation tracking were provided, and the stability of the presented protocol was demonstrated by utilizing the Lyapunov-like analysis theory. An interesting topic for future research is to deal with the case where the communication topology among followers can be directed. Another future research direction is to consider the distributed formation tracking problems for multiagent systems with nonlinear dynamics.

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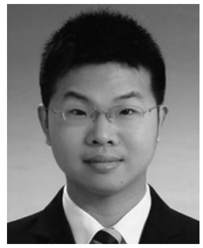

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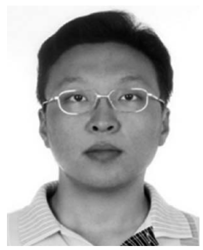

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